

$$y = e^{\lambda x} u$$

$$y' = \lambda e^{\lambda x} u + e^{\lambda x} u'$$

$$y'' = \lambda e^{\lambda x} u + \lambda^2 e^{\lambda x} u + \lambda e^{\lambda x} u' + \lambda e^{\lambda x} u' + e^{\lambda x} u''$$

حالتی از y در معادله

$$\lambda e^{\lambda x} u + \lambda^2 e^{\lambda x} u + \lambda e^{\lambda x} u' + \lambda e^{\lambda x} u' + e^{\lambda x} u'' - (\lambda^2 e^{\lambda x} u + \lambda e^{\lambda x} u') + \lambda e^{\lambda x} u = 0$$

ساده سازی

$$e^{\lambda x} u'' + \lambda e^{\lambda x} u = 0 \xrightarrow{\div e^{\lambda x}} u'' + \lambda u = 0 \rightarrow s^2 + \lambda = 0 \rightarrow s = \pm \sqrt{\lambda} i$$

$$u_h = (A \cos \sqrt{\lambda} x + B \sin \sqrt{\lambda} x) \xrightarrow{u = \frac{y}{e^{\lambda x}}}$$

$$y = e^{\lambda x} (A \cos \sqrt{\lambda} x + B \sin \sqrt{\lambda} x)$$

شرایط مرزی

$$\left\{ \begin{array}{l} y(0) = 0 \rightarrow 0 = 1(A) \rightarrow \boxed{A=0} \\ y'(0) = \sqrt{\lambda} \rightarrow y = e^{\lambda x} (B \sin \sqrt{\lambda} x) \rightarrow y' = \lambda e^{\lambda x} (B \sin \sqrt{\lambda} x) + \sqrt{\lambda} \cos \sqrt{\lambda} x B e^{\lambda x} \end{array} \right.$$

$$\sqrt{\lambda} = \sqrt{\lambda} B \rightarrow \boxed{B=1}$$

$$\rightarrow y = e^{\lambda x} (\sin \sqrt{\lambda} x) \xrightarrow{y(\sqrt{\lambda})} e^{\lambda} (\sin 1) \rightarrow \boxed{\frac{e^{\lambda} \sin 1}{\sqrt{\lambda}}}$$

(۴۲)

$$r = c(1 - \sin \theta)$$

$$r' = -c \cos \theta \rightarrow c = \frac{-r'}{\cos \theta}$$

$$\rightarrow r = \frac{-r'}{\cos \theta} (1 - \sin \theta)$$

$$r' = \frac{-r^2}{r'} \rightarrow r = \frac{+r^2 (1 - \sin \theta)}{r' \cos \theta} \rightarrow r' \cos \theta = \frac{r^2 (1 - \sin \theta)}{r}$$

$$r' \cos \theta = r(1 - \sin \theta) \rightarrow \frac{dr}{d\theta} \cos \theta = r(1 - \sin \theta) \rightarrow \frac{dr}{r} = \frac{1 - \sin \theta}{\cos \theta} d\theta$$

$$\ln r = \int \frac{1 - \sin \theta}{\cos^2 \theta} \times \cos \theta d\theta$$

$$\rightarrow \cos^2 \theta + \sin^2 \theta = 1 \rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\rightarrow \ln r = \int \frac{1 - \sin \theta}{(1 - \sin \theta)(1 + \sin \theta)} \cos \theta d\theta \rightarrow \ln r = \int \frac{\cos \theta}{1 + \sin \theta} \rightarrow \ln r = \ln(1 + \sin \theta) + \ln c$$

$$r = c(1 + \sin \theta) \rightarrow \text{گزیده}$$

(۴۳)

$$y' = p$$

$$y'' = p \frac{dp}{dy}$$

$$\rightarrow p \frac{dp}{dy} - 4p^2 - 3y = 0$$

$$z = p^2 \rightarrow 2pp' = z' \rightarrow pp' = \frac{z'}{2}$$

$$\rightarrow \frac{z'}{2} - 4z - 3y = 0 \xrightarrow{\times 2} z' - 8z = 6y \rightarrow \boxed{\text{خطی}}$$

ملاحظه

$$\begin{cases} e^{-\int -8 dy} = e^{8y} \\ \int e^{-8y} (6y) \Rightarrow \text{جزء جز} \Rightarrow \frac{(24y+3)e^{-8y}}{32} + C \end{cases}$$

$$\rightarrow z = \frac{-24y-3}{32} + Ce^{8y} \rightarrow z = p^2 \rightarrow p^2 = \frac{-24y-3}{32} + Ce^{8y}$$

شرایط مرزی: $1 = \frac{-3}{32} + C \rightarrow C = \frac{35}{32} \rightarrow$ ^ی ^{قادر} ^{بسیار} ^{کم} ^{است} ^{با} ^{توان} ^{ما} ^{یک} $\rightarrow \frac{-27}{32} + \frac{35}{32} e^1 = \frac{35e^1 - 27}{32}$ (۴۳)

$$F(s) = \tanh^{-1}\left(\frac{1}{s}\right)$$

$$\boxed{\text{طراحی}} \xrightarrow{\text{مشتق}} \tanh^{-1} u = \frac{u'}{1-u^2}$$

$$\frac{dF(s)}{ds} = \frac{-\frac{1}{s^2}}{1 - \frac{1}{s^2}} \rightarrow \dots$$

$$\frac{dF(s)}{ds} = \frac{-1}{s^2-1} \rightarrow -t f(t) = -\sinh t \rightarrow f(t) = \frac{\sinh t}{t} \rightarrow \underline{\underline{\text{مشتق}}}$$

(۴۵)

$$(2n+1)y'' - ny' + y = 0 \quad y(0) = 1$$

$$y'' \text{ ی مقدار برابر } n=0 \rightarrow y''(0) + y(0) = 0 \rightarrow y''(0) = -1 \quad y'(0) = 0$$

$$\textcircled{I} \quad x^3 \text{ ضرب } \rightarrow \frac{y^3}{3!}$$

$$2y''(0) + (2n+1)y''' - y' - ny'' + y' = 0$$

$$\rightarrow -2 + y''' - 0 - 0 + 0 = 0 \Rightarrow y''' = 2$$

$$\textcircled{I} \rightarrow \frac{2}{6} = \frac{1}{3} \rightarrow \underline{\underline{\text{دو برابر}}}$$